SPACECRAFT ANGULAR STATE ESTIMATION AFTER SENSOR FAILURE

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ABSTRACT

This work describes two algorithms for computing the angular rate and attitude in case of a gyro failure in a spacecraft (SC) with a special mission profile. The source of the problem is presented, two algorithms are suggested, an observability study is carried out, and the efficiency of the algorithms is demonstrated.

KEY WORDS

Attitude Determination, Angular Rate Computation, Kalman Filtering, Sensor Failure.

1. INTRODUCTION

Consider a spacecraft (SC) that has two two-axis gyroscopes, which are arranged such that one input axis of one of the gyros is aligned along the body x-axis and the other input axis is aligned along the body z-axis. The input axes of the second gyro are aligned along the body y and z-axes. If only one gyro fails then the rate about two body axes is still measured; namely, about the z-axis and one of the other two. The SC considered has also a vector measuring devise like a sun-sensor. It carries also a second vector-measuring device but it is assumed that this sensor failed too. The angular motion of the SC consists of two rotations as described in Fig. 1. One rotation, at $\dot{\xi}$ radian per second, is about a fixed line in inertial space, and the other, at λ radian per second, is about the satellite z-axis. The body z-axis is tilted by α radians with respect to some line fixed in inertial space. It is require that in the event
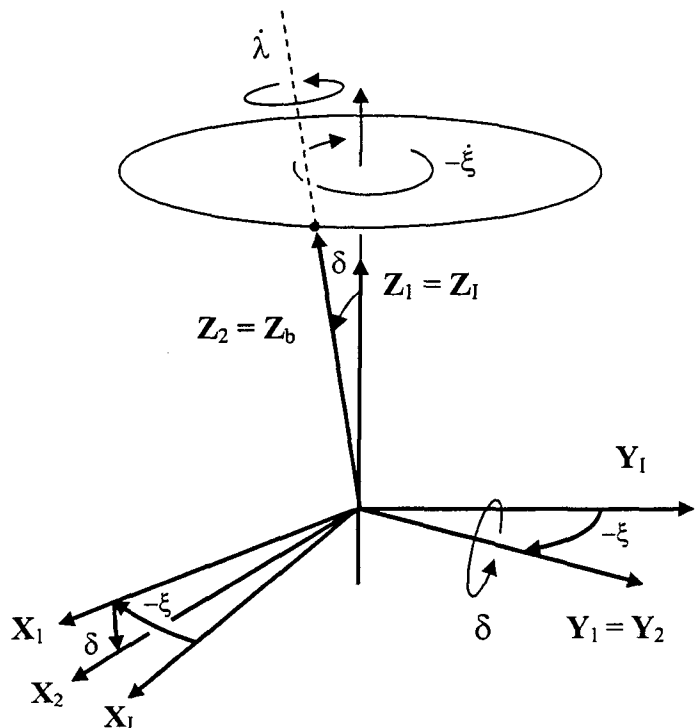

Fig. 1: The Geometry and Coordinate Systems Describing the Angular Motion Components of the Spacecraft.

of a failure of one of the gyros, the angular velocity and attitude of the SC can still be fully determined. There were two cardinal questions concerning such failure; namely, can the full angular rate vector and attitude be determined, and if it can be, how to do it. An answer to the first question is supplied by an observability check; that is, if the system state vector, containing the attitude and angular rate vector, is observable with one gyro and one vector measurement, then it is possible to estimate the rate and attitude of the SC. In this case what is left to do is to devise an algorithm that does it. These two questions are addressed in this work. As will be shown in the ensuing, the answer to these questions is positive. Two algorithms are presented in this work, one of which may supply the rate and attitude of the SC when one gyro fails.

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2. BODY ANGULAR KINEMATICS

The body angular rate and attitude definitions are presented in Fig. 1 in terms of three Euler angles. In order to describe the SC motion and the associated measurements we define four coordinate systems. The first system is the inertial coordinate system (designated by I). The center of this system is at the origin of the SC coordinates, its x-axis and z-axis point arbitrarily in inertial space. A rotation by the angle $-\xi$ about the z-axis yields a first interim coordinate system, which we denote by 1. A second rotation by the angle δ about the y-axis of this interim coordinate system yields the second interim coordinates, which we denote by 2. Finally, a rotation about the z-axis of the latter system by λ results in the body coordinates, which we denote by b. Note that, for simplicity, only the z-axis of the body system is shown in Fig. 1. The SC rotates about two axes. One rotation, at the rate of $-\dot{\xi}$, is about the Z_I axis, and the other, at the rate of $\dot{\lambda}$, is about the body z-axis. The sum of these rotation rates in body coordinates is as follows:

$$\omega_x = \dot{\xi}\sin\delta \cdot \cos\lambda + \dot{\delta}\sin\lambda$$
$$\omega_y = -\dot{\xi}\sin\delta \cdot \sin\lambda + \dot{\delta}\cos\lambda \qquad (1)$$
$$\omega_z = -\dot{\xi}\cos\delta + \dot{\lambda}$$

These rate components are inertial rates resolved in body coordinates. We note that since δ is designed to be constant the design value of the derivative of δ is:

$$\dot{\delta} = 0 \qquad (2)$$

The transformation matrix from inertial to body coordinates is:

$$D_b^I = \begin{bmatrix} c\xi c\delta c\lambda + s\xi s\lambda & -s\xi c\delta c\lambda + c\xi s\lambda & -s\delta c\lambda \\ -c\xi c\delta s\lambda + s\xi c\lambda & s\xi c\delta s\lambda + c\xi c\lambda & s\delta s\lambda \\ c\xi s\delta & -s\xi s\delta & c\delta \end{bmatrix} \qquad (3)$$

The corresponding quaternion can be extracted best when using the algorithm presented in [1].

The Vector Measurements
We need to determine the relationship between the vector measurements that are measured by a Sun Sensor (SS), for example, and the angles ξ, δ and λ that determine SC attitude. To do that we turn to Fig. 2, which depicts the orientation of the sun-sensor boresight in body coordinates. The SS reads the tangent of the angles α and β. We wish to express the readings of these tangents as functions of the angles ξ, δ and λ defined in Fig. 1. To do that we define $(Z_I)_b$ and $(Z_I)_I$. The column matrix $(Z_I)_b$ contains the components of the vector Z_I when resolved in the body coordinates, and, similarly, the column matrix $(Z_I)_I$ contains the components of the vector Z_I when resolved in the
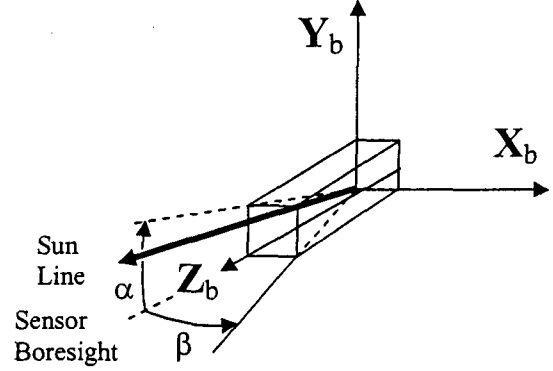

Fig. 2: Definition of the Sun Direction

inertial coordinates. We denote the components of the former by a_x, a_y and a_z. It is easy to see that the components of $(Z_I)_I$ are 0, 0, and 1. Thus

$$(Z_I)_b^T = \begin{bmatrix} a_x & a_y & a_z \end{bmatrix} \qquad (4.a)$$

and

$$(Z_I)_I^T = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \qquad (4.b)$$

Obviously

$$(Z_I)_b = D_b^I (Z_I)_I \qquad (5)$$

From the last three equations and Eq. (3) we obtain

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \times & \times & -s\delta c\lambda \\ \times & \times & s\delta s\lambda \\ & & c\delta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad (6)$$

hence

$$a_x = -s\delta c\lambda$$
$$a_y = s\delta s\lambda \qquad (7)$$
$$a_z = c\delta$$

therefore

$$\tan\beta = \frac{a_x}{a_z} = -\tan\delta \cdot c\lambda$$

$$\qquad (8)$$

$$\tan\alpha = \frac{a_y}{a_z} = \tan\delta \cdot s\lambda$$

Note that $\tan\alpha$ and $\tan\beta$ are independent of ξ. An inspection of Fig. 2 reveals that this is an expected result. Since $\tan\alpha$ and $\tan\beta$ are measured quantities, the last two equations contain only two unknowns, therefore we can solve for the angles δ and λ. For this reason even if the measurements contain errors we can still find these two angles by proper filtration as long as the measurements are unbiased. In order to fully know the SC attitude we need to know ξ too. If the gyros mounted on the inertial coordinates z-axis introduce no bias, then a proper integration of their measured outputs will yield ξ and thus we can determine the

SC attitude from the measurements obtained from these gyros and the SS. If the gyro measurements contain bias then we ask ourselves whether the system is observable; that is, whether the SC angular dynamics (Euler) equation may add information with which we can still observe ξ and thus completely determine the attitude. To answer this question we have to design an estimator and examine the observability of the system used by the estimator. This is done next.

3. THE FILTERS

III.1 Filter I

III.1.1 Dynamics Model

In this estimator (filter) we convert the SS measurements to a vector measurement, which is then connected to the quaternion of the SC. The dynamics model of Filter I was developed in [2]. Using

$$Q = \begin{bmatrix} q_4 & -q_3 & q_2 \\ q_3 & q_4 & -q_1 \\ -q_2 & q_1 & q_4 \\ -q_1 & -q_2 & -q_3 \end{bmatrix} \qquad (9)$$

define the system dynamics as

$$\begin{bmatrix} \dot{\omega} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} I^{-1}[(I\omega + h)\times] & 0 \\ \frac{1}{2}Q & 0 \end{bmatrix}\begin{bmatrix} \omega \\ q \end{bmatrix} + \begin{bmatrix} I^{-1}(T - \dot{h}) \\ 0 \end{bmatrix} + \begin{bmatrix} w_\omega \\ w_q \end{bmatrix} \quad (10)$$

where $[g\times]$ is the cross product matrix of a general vector g, w_ω accounts for the inaccuracies in the modeling of the SC angular dynamics, and w_q accounts for modeling errors of the quaternion dynamics. Both w_ω and w_q are zero mean white noise vectors that are uncorrelated with one another.

III.1.2 Measurement Equations

The measurements are the tangent of the angles α and β, measured by the SS and shown in Fig. 4, and the z and either the x or the y component of the body rate measured by one of the SC gyros. The measured quantities are contaminated by noise so they are not the true values but rather, α_m, β_m, $\omega_{z,m}$ and either $\omega_{x,m}$ or $\omega_{y,m}$, depending which gyro failed.

III.1.2.1 Sun Sensor Measurement
Using the two measured angles we compute a unit vector in the assumed sun direction as follows [4, p. 226]

$$b_m = \frac{1}{\sqrt{\tan\alpha_m^2 + \tan\beta_m^2 + 1}}\begin{bmatrix} \tan\beta_m \\ \tan\alpha_m \\ 1 \end{bmatrix} \qquad (11)$$

In reality $\tan\beta_m$ and $\tan\alpha_m$ contain noise; however, in the filter we approximate the b_m measured vector as the true vector to the sun and a simple additive zero mean white noise vector, v_b; that is,

$$b_m = b + v_b \qquad (12)$$

where b is the true unit vector in the true direction to the sun. The logic behind this simplification is that if in tests the filter yields satisfying results then the approximation is justified. Let r denote the measured sun-vector expressed in the reference coordinate system (this vector is taken from the almanac) then the relationship between the two vectors is expressed by

$$b = Dr \qquad (13)$$

where D is the direction cosine matrix that transforms vectors from the reference to the body coordinates. From the last two equations we obtain

$$b_m = Dr + v_b \qquad (14)$$

It is well known that D is the following function of the quaternion elements [3, p. 414]

$$D = \begin{bmatrix} q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2(q_1 q_2 + q_3 q_4) & 2(q_1 q_3 - q_2 q_4) \\ 2(q_1 q_2 - q_3 q_4) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2(q_2 q_3 + q_1 q_4) \\ 2(q_1 q_3 + q_2 q_4) & 2(q_2 q_3 - q_1 q_4) & -q_1^2 - q_2^2 + q_3^2 + q_4^2 \end{bmatrix}$$
$$\dots (15)$$

then using this relationship between D and q, Eq. (14) can be written as

$$b_m = \begin{bmatrix} 0 & H_b(r,q) \end{bmatrix}\begin{bmatrix} \omega \\ q \end{bmatrix} + v_b \qquad (16)$$

were

$$H_b(r,q) = \begin{bmatrix} q_1 r_1 + q_2 r_2 + q_3 r_3 & -q_2 r_1 + q_1 r_2 - q_4 r_3 & -q_3 r_1 + q_4 r_2 + q_1 r_3 & q_4 r_1 + q_3 r_2 - q_2 r_3 \\ q_2 r_1 - q_1 r_2 + q_4 r_3 & q_1 r_1 + q_2 r_2 + q_3 r_3 & -q_4 r_1 - q_3 r_2 + q_2 r_3 & -q_3 r_1 + q_4 r_2 + q_1 r_3 \\ q_3 r_1 - q_4 r_2 - q_1 r_3 & q_4 r_1 + q_3 r_2 - q_2 r_3 & q_1 r_1 + q_2 r_2 + q_3 r_3 & q_2 r_1 - q_1 r_2 + q_4 r_3 \end{bmatrix}$$
$$\dots(17)$$

This is the measurement equation associated with the SS measurement.

III.1.2.2 Gyro Measurement
Let us assume that the gyro that measures rates along the x and z axes is the gyro that is still operating. The x and z axes rate measurements can be written as

$$\omega_{x,m} = \omega_x + v_{\omega x}$$
$$\omega_{z,m} = \omega_z + v_{\omega z} \qquad (18)$$

where ω_x and ω_z are the true rates, and $v_{\omega x}$ and $v_{\omega z}$ are *scalar* zero-mean white measurements noise. Define

$$H_\omega = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (19)$$

then from the last two equations we obtain

$$\begin{bmatrix} \omega_{x,m} \\ \omega_{z,m} \end{bmatrix} = \begin{bmatrix} H_\omega & 0 \end{bmatrix}\begin{bmatrix} \omega \\ q \end{bmatrix} + \begin{bmatrix} v_{\omega x} \\ v_{\omega z} \end{bmatrix} \qquad (20)$$

We denote the measurement vector by ω_m where, clearly, $\omega_m^T = \begin{bmatrix} \omega_{x,m} & \omega_{z,m} \end{bmatrix}$. Similarly we denote the noise vector as by v_ω where $v_\omega^T = \begin{bmatrix} v_{\omega x} & v_{\omega z} \end{bmatrix}$. Obviously, if the other gyro is the one that is still operating, then the x component in Eqs. (18 – 20) has to be replaced by the y.

III.1.2.3 The Combined measurement
Combining Eqs. (21) and (24) we obtain

$$\begin{bmatrix} \mathbf{b}_m \\ \boldsymbol{\omega}_m \end{bmatrix} = \begin{bmatrix} 0 & H_b(\mathbf{r},\mathbf{q}) \\ H_\omega & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{q} \end{bmatrix} + \begin{bmatrix} \mathbf{v}_b \\ \mathbf{v}_\omega \end{bmatrix} \qquad (21)$$

This is the combined measurement model for simultaneous measurement of the operating two-axes gyro and the SS.

III.2 Filter II
III.2.1 Dynamics Model
In this filter we make use of the kinematics rule governing the rate of change of the sun vector. It is known that

$$D\dot{\mathbf{r}} = \dot{\mathbf{b}} + \boldsymbol{\omega} \times \mathbf{b} \qquad (22.a)$$

hence

$$\dot{\mathbf{b}} = [\mathbf{b} \times]\boldsymbol{\omega} + D\dot{\mathbf{r}} \qquad (22.b)$$

As mentioned earlier, the vector $\mathbf{r}$ is known from the almanac, therefore $\dot{\mathbf{r}}$ is known too. Consequently we can treat $D\dot{\mathbf{r}}$ as a deterministic forcing function. In our case $\mathbf{r}$ is the vector from the SC to the sun. For the SC trajectory this vector changes extremely slowly, therefore we can neglect the term in $\dot{\mathbf{r}}$ and use

$$\dot{\mathbf{b}} = [\mathbf{b} \times]\boldsymbol{\omega} \qquad (22.c)$$

Augmenting Eq. (22.c) with the dynamics of Filter I yields

$$\begin{bmatrix} \dot{\boldsymbol{\omega}} \\ \dot{\mathbf{q}} \\ \dot{\mathbf{b}} \end{bmatrix} = \begin{bmatrix} I^{-1}[(I\boldsymbol{\omega}+\mathbf{h})\times] & 0 & 0 \\ \tfrac{1}{2}Q & 0 & 0 \\ [\mathbf{b}\times] & 0 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{q} \\ \mathbf{b} \end{bmatrix} + \begin{bmatrix} I^{-1}(T-\dot{\mathbf{h}}) \\ 0 \\ D\dot{\mathbf{r}} \end{bmatrix} + \begin{bmatrix} \mathbf{w}_\omega \\ \mathbf{w}_q \\ \mathbf{w}_b \end{bmatrix} \qquad (23)$$

Remark: The state vector in Eq. (23) includes the sun vector $\mathbf{b}$ even though $\mathbf{b}$ is a measurement. Indeed, formally, in order to compute the rate and attitude one can numerically differentiate one of the components of $\mathbf{b}$, and use it in one of the following equations (which are the components of Eq. (22.c))

$$\dot{b}_x = \omega_z b_y - \omega_y b_z$$
$$\dot{b}_y = \omega_x b_z - \omega_z b_x \qquad (24)$$
$$\dot{b}_z = \omega_y b_x - \omega_x b_y$$

together with the gyro-measured ω_z, and either ω_x or ω_y (depending which gyro is intact) to solve for the missing component of $\boldsymbol{\omega}$. Then use the full $\boldsymbol{\omega}$ as 'measurements' in an estimator whose dynamics model consists of the first two matrix equations in Eq. (23). This, however, requires numerical differentiation, which introduces noise. To avoid differentiation we apply the *estimation approach* [4] according to which we add to Eqs. (22.c) the differential equations of the variables that we want to estimate, thereby forming Eq. (23), and then use a Kalman filter to estimate those variables.

III.2.2 Measurement Equations
The measurement equations of Filter II are quite simple. The vector measurement of Eq. (16) is replaced by the

following simple equation.

$$\mathbf{b}_m = \begin{bmatrix} 0 & 0 & I \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{q} \\ \mathbf{b} \end{bmatrix} + \mathbf{v}_b, \qquad (25)$$

Eq. (20) stays the same; therefore, the augmented measurement equation for Filter II becomes

$$\begin{bmatrix} \mathbf{b}_m \\ \boldsymbol{\omega}_m \end{bmatrix} = \begin{bmatrix} 0 & 0 & I \\ H_\omega & 0 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{q} \\ \mathbf{b} \end{bmatrix} + \begin{bmatrix} \mathbf{v}_b \\ \mathbf{v}_\omega \end{bmatrix} \qquad (26)$$

4. OBSERVABILITY TESTS

Before carrying on an observability test it is necessary to specify the kind of filter that was used in this analysis. The filter that was used here is the PSELIKA (Pseudo-Linear Kalman) Filter [5]. The idea on which this filter is based is as follows. Suppose that the non-linear dynamics and measurement equations of a non-linear system can be decomposed in the following way

$$\dot{x} = F(x)x + Bu + w \qquad (27.a)$$
$$y = H(x)x + v \qquad (27.b)$$

If $F(\hat{x})$ is close enough to $F(x)$ and $H(\hat{x})$ is close enough to $H(x)$ (note that we do not assume that $\hat{x}$ is necessarily close to x) then instead of the unknown matrices $F(x)$ and $H(x)$ we can use respectively $F(\hat{x})$ and $H(\hat{x})$, which are known along the estimation process, and apply the linear Kalman filter algorithm to the measurements using the models

$$\dot{x} = F(\hat{x})x + Bu + w \qquad (27.c)$$
$$y = H(\hat{x})x + v \qquad (26.d)$$

In the observability analysis we use $F(x)$ and $H(x)$ where x is taken from simulation. The logic of this test is as follows. If when using $F(x)$ and $H(x)$ in the analysis the latter indicates that the system is not observable under these favorable conditions then there is no sense to try to estimate the state vector x of the system. If, on the other hand, the analysis indicate that the system is observable, then observability and therefore estimability are not assured, but it make sense to design an estimator, try to estimate x and see if this could indeed be done.

The observability of the system models used by the two filters was examined using the following analysis. Consider Φ_k, the transition matrix, which corresponds to F, the dynamics matrix, of either Eq. (10) or Eq. (23). This matrix transforms the system state vector at time t_k, to x_{k+1}, the state at time t_{k+1}. If at a certain time-point, t_m, the initial state, denoted by x_0, can be computed, then, for our purposes, the system is observable. Adopting the common approach to the proof

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of complete observability of a discrete linear system, we express the first m measurements as follows

$$\mathbf{y}_j = \mathbf{H}_j(\mathbf{x}_j) \left[\prod_{i=0}^{j-1} \Phi_i(\mathbf{x}_i) \right] \mathbf{x}_0 \quad j = 0, 1, \ldots, m \quad (28)$$

where $\Phi_0(\mathbf{x}_0) = \mathbf{I}_n$. Form the matrix equation

$$\begin{bmatrix} \mathbf{y}_0 \\ \mathbf{y}_1 \\ \mathbf{y}_2 \\ \cdot \\ \cdot \\ \mathbf{y}_m \end{bmatrix} = \begin{bmatrix} \mathbf{H}_0(\mathbf{x}_0) \\ \mathbf{H}_1(\mathbf{x}_1)\Phi_0(\mathbf{x}_0) \\ \mathbf{H}_2(\mathbf{x}_2)\Phi_1(\mathbf{x}_1)\Phi_0(\mathbf{x}_0) \\ \cdot \\ \cdot \\ \mathbf{H}_m(\mathbf{x}_m)\Phi_{m-1}(\mathbf{x}_{m-1})\cdots\Phi_0(\mathbf{x}_0) \end{bmatrix} \mathbf{x}_0 \quad (29)$$

(Note that in Filter II H is not a function of the state. That is, the measurement equation is linear from the start.) If there are n independent rows in the right hand side matrix (the observability matrix) in Eq. (29), where n is the number of states, then, of course, $\mathbf{x}_0$ can be solved for, hence the state is observable.

The observability test of Filter I reveals that the rank of the observability matrix for this filter becomes 7, and since the filter state is 7 too, the system is observable. It means that there are good chances that Filter I can estimate both the missing angular rate component as well as the attitude of the SC when one gyro fails. We note though that the measurement equation of Filter I is only pseudo-linear and not truly linear. This may cause a problem even though the observability test indicates that the state is completely observable. The observability matrix of Filter II becomes 6 and stay at 6 whereas the size of the state vector is 9. This indicates that the system is not completely observable. It means that we cannot fully estimate both the missing rate component and the attitude. Eqs. (24) and the discussion that follows that equation indicate that the missing rate can be observed. Therefore it is concluded that it is the attitude, which cannot be fully estimated when Filter II is used.

5. TEST RESULTS

Since Filter II can estimate only the rate vector, we present here only simulation runs that were carried out using Filter I. Simulations were run for different initial conditions and noise level using Filter I. Plots that resulted from a typical run are presented in Figs. 3 - 8. For this run the initial ξ angle was 60 degrees, δ was 35 degrees and λ was 0. The initial estimates were zero for all three angles. The true Euler-angle rates were $2\cdot(2\pi)/3600$ rad/sec for $\dot{\xi}$, zero for $\dot{\delta}$ and $0.928\cdot(2\pi)/60$ rad/sec for $\dot{\lambda}$. According to Eq. (1), using these initial rates and angles, the true initial angular rate components were as follows: $\omega_x = 0.002$ rad/sec $\omega_y = 0$ rad/sec and $\omega_z = 0.094$ rad/sec. The estimated values of the angular velocity were zero for all three components. The one-sigma gyro measurement noise was 0.01 deg/sec and that of the SS was 0.25 degrees. In each figure we see the true state and superimposed on it is the estimate of the state. Underneath this plot there is a plot of the estimation error. In this run ξ is estimated quite well, but this is not always the

case. The observability of this angle is quite poor. Filter II was also run but, as expected from the observability test, the three attitude angles were not observable.
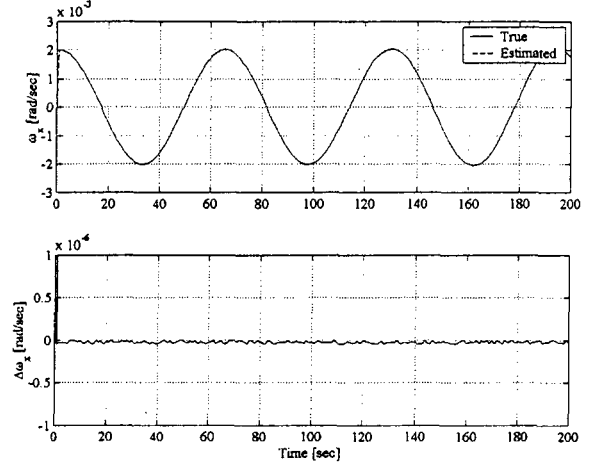

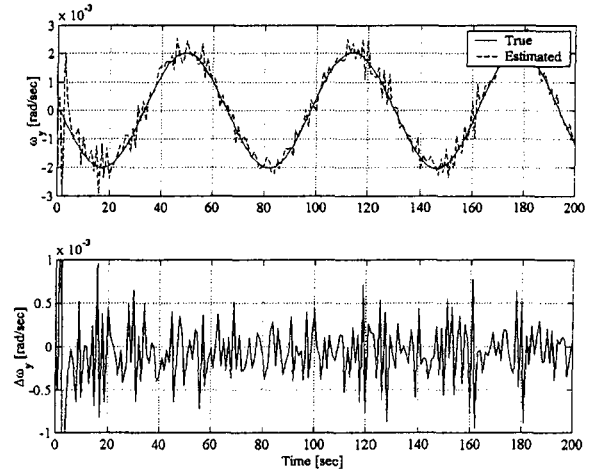**Fig. 3:** The x-component of the angular velocity and its estimation error.

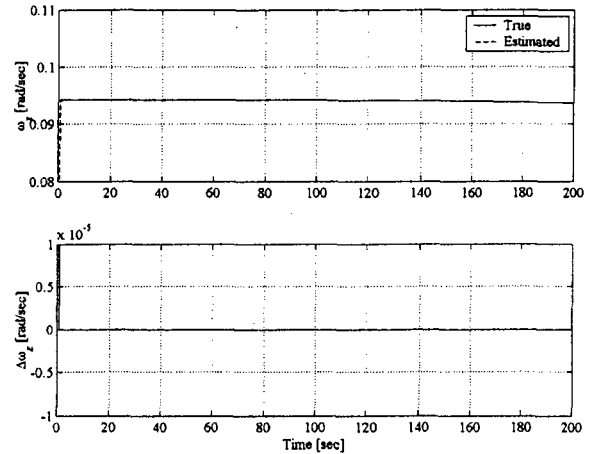**Fig. 4:** The y-component of the angular velocity and its estimation error.

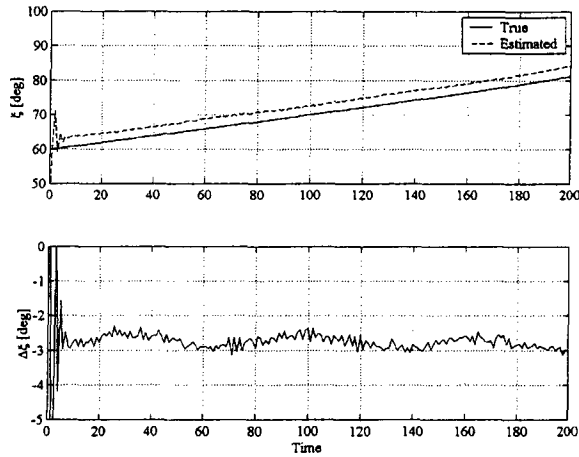**Fig. 5:** The z-component of the angular velocity and its estimation error.
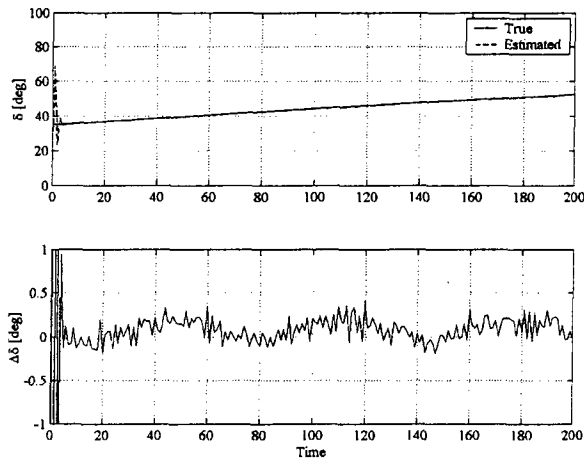
Fig. 6: The true and estimated ξ angle.

Fig. 7: The true and estimated δ angle.

Fig. 8: The true and estimated λ angle.

6. CONCLUSIONS

In this paper we considered a spacecraft that uses two two-degrees of freedom gyros and two vector measuring devices like a SS for measuring the rate and the attitude of the spacecraft. We wanted to investigate the possibility of estimating the spacecraft full rate vector and attitude if one gyro and one vector-measuring device fail simultaneously. In our analysis we proposed two filters for estimating the desired variables and carried on an observability study for both filters. We found that the variables were completely observable when one of the filters was used, and was not completely observable when the other filter was used. We tested our conclusions through simulations and found that indeed Filter I can estimate all states however, one of the attitude angles was poorly observable. As expected, Filter II was unable to estimate the attitude angles.
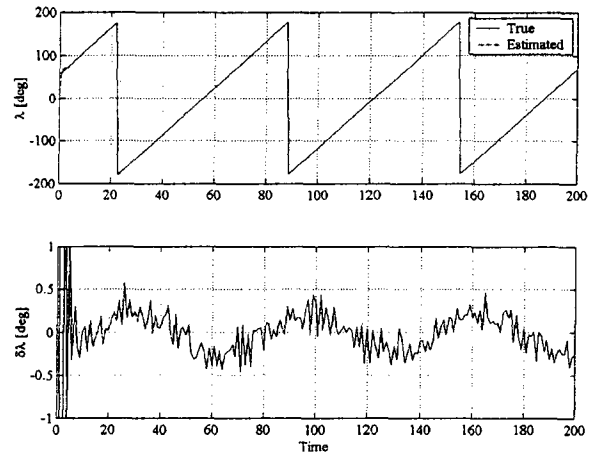
7. ACKNOWLEDGEMENT

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